8. Yu. A. Buyevich, "Bemerkung über die Konstruktion von Modellen für wandnahe Turbulenz," Z. Angew. Math. Mech., 49, No. 6, 372-374 (1969).
9. E. R. Van Driest, "On turbulent flow near a wall," J. Aeronaut. Sci., 23, No. 11, 1007-1011 (1956).
10. W. Szablewski, "Über turbulente Scherströmungen," Monatsberichte Deutsch. Akad. Wiss., 9, No. 8, 557-573 (1967).

PRECIPITATION OF A CLOUD OF HEATED PARTICLES ON A HORIZONTAL PLANE

G. M. Makhviladze and O. I. Melikhov UDC 532.529:536.46

The precipitation of a cloud of heated monodisperse particles in a field of external force is numerically investigated.

The nature of the motion of a cloud of particles in an infinite medium under the action of an external force (gravity) depends on the degree of hydrodynamic interaction between the particles, which is mediated by the carrier phase [1-3]. If the particle concentration in the cloud is low, each particle moves like a solitary particle, independently of the others (the "filtration" regime). If the particle concentration is sufficiently high this regime gives way to the "entrainment" regime, where the medium between the particles is involved in the motion; the assembly of particles moves at a speed that exceeds the speed of fall of a single particle.

In [4, 5] the motion and precipitation of a cloud possessing plane symmetry (in a direction perpendicular to the action of the external force one of the dimensions of the cloud was much greater than the other) were examined. A numerical solution of the unsteady two-dimensional equations of motion of a two-phase medium showed that the interaction of the cloud with the precipitation surface depends significantly on the regime of motion. In the filtration regime all the particles move in straight trajectories, perpendicular to the precipitation plane, and fall on this plane in the region of initial projection of the cloud; the final distribution of the precipitated particles does not depend on the initial height of the cloud. Entrainment motion of the cloud produces a large-scale vortical flow of the carrier medium in the form of two symmetric cylindrical eddies, which increase in size (the analog of this solution in the case of axial symmetry is a vortical ring — a torus). In the plane of symmetry the gas moves downward in the direction of the external force and on the periphery it rises. The particles are involved in this motion and become concentrated in the cores of the eddies; thus, most of the particles move in directions perpendicular to that of the external force. This effect becomes more pronounced as the cloud approaches the precipitation plane owing to the spreading of the gas along it, which causes additional sideways transport of particles. This leads to precipitation of some particles at great distances from the plane of symmetry, exceeding the initial radius of the cloud.

In [4, 5] the precipitation process was considered in the isothermal case, where the gas and particle temperatures are equal. In this paper we examine, in a plane formulation, the precipitation of a cloud of particles in the nonisothermal case, where the initial particle temperature exceeds the temperature of the surrounding gas. At the initial time a cold gas, in static equilibrium in a field of external force, contains a motionless cloud of heated solid or liquid spherical particles (a monodisperse aerosol). In describing the motion of the disperse medium we adopt the usual assumptions of mechanics of heterogeneous media [6], and regard the gas and particles as two interpenetrating and interacting continua. We consider systems in which the volume fraction of the particles and the ratio of the true gas and particle densities are small; collisions, breakup, and evaporation of the particles are insignificant; viscous dissipation of energy is negligible; the temperature is assumed constant through-

Institute of Mechanics Problems, Academy of Sciences of the USSR, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 45, No. 2, pp. 238-244, August, 1983. Original article submitted May 24, 1982.

 0022-0841/83/4502-0882$07.50

out the volume of the particle; the precipitation surface is thermally insulated; the dynamic viscosity and thermal conductivity are constant.

When the above assumptions are made the plane unsteady motion of the two-phase medium in a Cartesian coordinate system (x, y), whose origin is under the centroid of the cloud and whose y axis is normal to the cloud and opposite to the external force vector, is represented by the following equations:

$$\rho_1 \frac{d_1\mathbf{U}_1}{dt} = -\frac{1}{\gamma M^2}\nabla P + \rho_1\mathbf{g} - \mathbf{f} + \frac{1}{\mathrm{Re}}\left[\Delta\mathbf{U}_1 + \frac{1}{3}\nabla(\nabla\mathbf{U}_1)\right],$$

$$\frac{d_1\rho_1}{dt} = -\rho_1(\nabla\mathbf{U}_1), \quad P = \rho_1\theta_1, \tag{1}$$

$$\rho_1\frac{d_1\theta_1}{dt} = \frac{\gamma}{\mathrm{RePr}}\Delta\theta_1 - (\gamma-1)P(\nabla\mathbf{U}_1) + q,$$

$$\frac{d_2\rho_2}{dt} = -\rho_2(\nabla\mathbf{U}_2), \ \rho_2\frac{d_2\theta_2}{dt} = -\gamma_1 q, \ \rho_2\frac{d_2\mathbf{U}_2}{dt} = \rho_2\mathbf{g} + \mathbf{f}, \tag{2}$$

where

$$q = 6\frac{\gamma\varepsilon}{\delta}\frac{\mathrm{Nu}}{\mathrm{Re}_p^0\mathrm{Pr}}\rho_2(\theta_1 - \theta_2) \quad (\mathrm{Nu} = 2 + 0.6\mathrm{Pr}^{1/3}\mathrm{Re}_p^{1/2}),$$

$$\mathbf{f} = \frac{3}{4}\frac{\varepsilon}{\delta}c_d\frac{\rho_1\rho_2}{1-\alpha_2^0 n}|\mathbf{U}_1 - \mathbf{U}_2|(\mathbf{U}_1 - \mathbf{U}_2), \ c_d = \frac{24}{\mathrm{Re}_p}(1 + 0.158\,\mathrm{Re}_p^{2/3}),$$

$$M^2 = Rg/\gamma R_0 T_0, \ \mathrm{Re} = (R^3 g)^{1/2}\rho_{10}/\eta, \ \mathrm{Pr} = c_p\eta/\lambda, \tag{3}$$

$$\gamma = c_p/c_v, \ \gamma_1 = c_p/c_2, \ \varepsilon = \rho_{10}/\rho_2^0, \ \delta = d/R,$$

$$\mathrm{Re}_p = \mathrm{Re}_p^0\frac{\rho_1}{1-\alpha_2^0 n}|\mathbf{U}_1 - \mathbf{U}_2|, \ \mathrm{Re}_p^0 = \frac{d(Rg)^{1/2}\rho_{10}}{\eta},$$

$$\mathbf{g} = (0, -1), \ \nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y}, \ \frac{d_i}{dt} = \frac{\partial}{\partial t} + (\mathbf{U}_i\nabla), \ \Delta = (\nabla\nabla),$$

with initial and boundary conditions

$$t = 0, \ \theta_1 = 1, \ \theta_2 = 1 + \theta_p, \ \mathbf{U}_i = 0, \ P = \exp(-\gamma M^2 y),$$

$$n = \exp[-x^2 - (y-H)^2], \ \rho_2 = n\alpha_2^0/\varepsilon \ (\alpha_2^0 = n_0\pi d^3/6), \tag{4}$$

$$x = 0, \ u_1 = 0, \ \partial v_1/\partial x = \partial\rho_1/\partial x = \partial\theta_1/\partial x = 0,$$

$$y = 0, \ \mathbf{U}_1 = 0, \ \partial\theta_1/\partial y = 0, \tag{5}$$

$$x^2 + y^2 \to \infty, \ \partial P/\partial y = -\gamma M^2 P, \ \theta_1 = 1, \ \mathbf{U}_1 = 0.$$

Problem (1)-(5) is written in dimensionless variables; the characteristic length, velocity, time, density, temperature, pressure, and particle concentration scales are R, $\sqrt{Rg}$, $\sqrt{R/g}$, ρ_{10}, T_0, $\rho_{10}R_0T_0$, and n_0. The subscript indicates the phase: i = 1 is the gas, i = 2 is the particles. In the calculations we used the following values of parameters: $M^2 = 0.75\cdot10^{-3}$, Pr = 1, $\gamma = 1.4$, $\gamma_1 = 1$, $\varepsilon = 10^{-3}$, Re = 29.05, $\delta = 3.3\cdot10^{-5}-1.4\cdot10^{-4}$, $\alpha_2^0 = 10^{-5}-10^{-2}$, H = 1.5-10, $\mathrm{Re}_p^0 = 6.5\cdot10^6\cdot\delta = 214.5-910$, $\theta_p = 0-5$. Here Re and Re_p^0 are assigned independently; the "external" Reynolds number is based on the effective turbulent viscosity, and the flow over the particle is assumed to be laminar. The equations of motion of the gas (1) were integrated by the method in [7, 8] and the particle equations (2) were integrated by a longitudinal–transverse scheme [9]. The method of solving problems of this type is described in [4].

At an early stage of the process the temperatures level out due to interphase heat transfer: The gas between the heated particles is heated by conduction and is set in motion. As

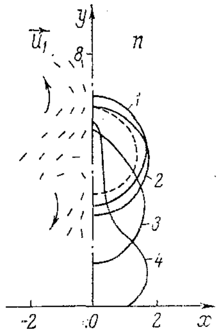

Fig. 1

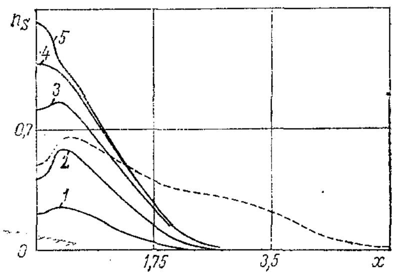

Fig. 2

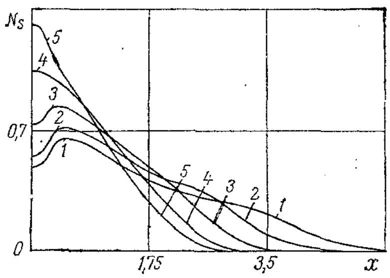

Fig. 3

Fig. 1. Evolution of a cloud of heated particles ($H = 5$, $\theta_p = 2$, $\delta = 6.67 \cdot 10^{-5}$, $\alpha_2^0 = 10^{-3}$); the $n = 0.1$ lines in the region $x > 0$ are shown: the dashed line is $t = 0$; 1) $t = 0.55$; 2) 1.84; 3) 5.52; 4) 9.2; the gas speed distribution at $t = 3.7$ in the region $x < 0$ is shown.

Fig. 2. Instantaneous particle distributions on precipitation plane, $H = 3.2$, $\theta_p = 2$, $\delta = 6.67 \cdot 10^{-5}$, $\alpha_2^0 = 10^{-3}$: 1) $t = 7.4$; 2) 9.2; 3) 11.1; 4) 12.9; 5) [coincides with $N_s(x)$] 18.4; dashed line — $N_s(x)$ at $\theta_p = 0$.

Fig. 3. Final particle distributions on precipitation plane in relation to initial temperature difference, $H = 3.2$, $\delta = 6.67 \cdot 10^{-5}$, $\alpha_2^0 = 10^{-3}$; 1) $\theta_p = 0$; 2) 0.5; 3) 1; 4) 1.5; 5) 2.

a result the dynamics of the settling cloud and the precipitation characteristics differ significantly from the isothermal case. These new effects arise if the cloud moves in the entrainment regime, which is considered below. If the particle concentration is sufficiently low (filtration regime) the motion of the cloud resembles the isothermal case — the heating of the gas is insignificant.

The effect of thermal processes on the evolution of a cloud of particles settling on a flat surface is shown in Fig. 1 (the picture is completely symmetric relative to the plane $x = 0$). The gas, heated by the thermal energy stored in the particles, expands in all directions in the (x, y) plane from the center of the cloud and carries the particles with it. This leads to a sharp increase in size (spread) of the cloud (curve 1) and to a reduction of the mean particle concentration. The relative increase in cloud radius in this case is $\sigma = 1.21$. In the initial stage of the process the particles descend under the action of the external force and are simultaneously transported by the expanding gas. The characteristic time of temperature equalization, defined as the time for reduction of the difference in the maximum (in space) temperatures of the two phases to $0.1\theta_p$, is $t_1 = 0.6$. After this time the expansion of the gas ceases (curve 1) and the particles move only downward (curve 2).

Large-scale (about the same size as the cloud) vortical motions of different physical nature then begin to form in the gas. At the bottom of the cloud, as in the isothermal case, a symmetric two-eddy flow, due to precipitation of the particles, is formed. At the top of the cloud, the development of thermal convection in the field of external force, as in the

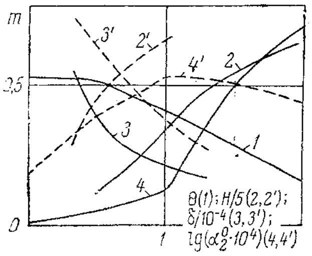

Fig. 4. Scattering coefficient as function of: 1) temperature ($H = 3.2$, $\delta = 6.67\cdot10^{-5}$, $\alpha_2^0 = 10^{-3}$); 2) initial cloud height ($\theta_p = 2$, $\delta = 6.67\cdot10^{-5}$, $\alpha_2^0 = 10^{-3}$); 3) particle diameter ($H = 3.2$, $\theta_p = 1.5$, $\alpha_2^0 = 10^{-3}$); 4) particle volume fraction ($H = 3.2$, $\theta_p = 2$, $\delta = 6.67\cdot10^{-5}$); the dashed lines are the corresponding curves for $\theta_p = 0$.

rising of hot volumes of gas ("thermals") [10], gives rise to another two symmetric eddies, which we will call "thermal" eddies, since they appear only in the nonisothermal case. The heated gas near the plane of symmetry at the top of the cloud moves upward. The time of onset of thermal convection can be estimated if the size of the region occupied by the heated gas ($2\sigma R$) and its characteristic speed of ascent $[2\sigma Rg(\langle\theta\rangle-1)]^{1/2}$ are known [10]. In dimensionless units $t_c' = [2\sigma/(\langle\theta\rangle-1]^{1/2} = 1.55$ ($\sigma = 1.21$, $\langle\theta\rangle = 2$), which gives for the time of onset of convection from the start of the process $t_c = t_1 + t_c' = 2.15$. Large-scale vortical motion due to the fall of the particles develops in a time t_s, which is approximately equal to the time in which the cloud falls through a distance equal to its radius. Estimating the speed of the cloud as the steady speed of fall of a single particle $\omega = 0.31$ [ω is determined by integrating the equation of motion of a single particle with due allowance for the relation $c_d(Re_p)$ from (3); the speed attains a steady value in time $t = 0.54$], we find $t_s \sim 1/\omega = 3.2$. The values of t_c and t_s agree with the results of calculation.

The formation of the four-eddy configuration composed of two ascending thermal eddies and two descending precipitation eddies is shown on the left-hand side of Fig. 1. The upward convective movement of the gas prevents the fall of the particles, the spread of the cloud, and sideways transport of the particles, with the result that the lines of equal concentration are produced in the vertical direction (curves 3 and 4). With the passage of time the thermal eddies leave the investigated region, and all the particles settle on the horizontal plane.

The special features of the motion of the cloud due to the initial temperature difference between the phases lead to a change in the instantaneous and final particle surface-concentration distributions, which are given by the relations:

$$n_s(x, t) = -\int_0^t n(x, 0, t')v_2(x, 0, t')\,dt',\quad N_s(x) = n_s(x, \infty).$$

In the isothermal case these distributions are characterized by two maxima, situated at points equidistant from the plane $x = 0$, and a minimum in the plane of symmetry, which can be attributed to the nature of the motion of the cloud in the entrainment regime. As Fig. 2 shows, when $\theta_p > 0$ the precipitation curves have initially the same shape as in the isothermal case. Subsequently, however, because of the vertical extension of the cloud, due to the development of thermal convection in the gas, the particles begin to settle mainly in the region near $x = 0$; the maximum of precipitated particles is shifted to the plane of symmetry.

The effect of nonisothermality on the final particle surface-concentration distributions is shown in Fig. 3. The greater the initial temperature difference, the more intense the convective flow of gas that leads to precipitation of particles near the plane of symmetry. In the isothermal case and for small θ_p we obtain two-humped final distributions (curves 1-3 correspond to them), whereas an increase in θ_p leads to a single concentration maximum in the plane of symmetry (curves 4 and 5).

A quantitative index of the precipitation process is the scattering coefficient m of the cloud on the precipitation plane, which is equal to the fraction of particles precipitated on the plane outside the initial projection of the cloud [4, 5]. The changes in the scattering coefficient in the nonisothermal case are due to two effects affecting the movements of the carrier medium; the spread of the cloud and the onset of thermal convection. The former promotes scattering of the cloud, whereas the latter causes the particles to concentrate near the axis of symmetry.

We compare the plots of scattering coefficient against initial cloud height, particle diameter, initial temperature drop, and particle volume fraction in the isothermal and nonisothermal cases (Fig. 4).

As Fig. 3 indicates, the scattering coefficient should decrease with increase in θ_p. The corresponding relation is shown by curve 1 in Fig. 4. As the cloud ascends its scattering increases (curves 2 and 2'), since with increase in H the precipitation eddies increase in size as they approach the plane. Reduction of the particle diameter with the other parameters fixed leads to their entrainment in the vortical motion and to scattering of the particles; the coefficient m increases in this case (curves 3 and 3'). For heated particles scattering for all H and δ is less than for cold particles. At large δ curves 3 and 3' will intersect in the region $m \approx 0$ owing to transition to the filtration regime.

In the isothermal case when the initial particle concentration is low the filtration regime occurs (curve 4') and almost all the particles are precipitated in the region of the initial projection of the cloud on the precipitation plane. When the volume fraction of the disperse phase is large gasdynamic interaction between the particles increases, there is a transition to the entrainment regime, and the scattering coefficient increases. In the case of large α_2^0 the cloud reaches the surface so quickly that the developing transverse motions of the carrier medium do not manage to transport the particles over great distances, and m decreases.

As distinct from the isothermal case, when the initial temperature of the particles is sufficiently high the function $m(\alpha_2^0)$ is monotonic in the investigated range of parameters (curve 4). Its increase at values of α_2^0 corresponding to the fall of the isothermal curve is due to the great spread of the cloud, which increases with increase in total thermal energy of the disperse phase in direct proportion to α_2^0. We note that when $\alpha_2^0 > 3.5 \cdot 10^{-3}$ the nonisothermal scattering coefficient exceeds the isothermal coefficient, as distinct from the other relations shown in Fig. 4. This can be attributed to the greater effect of spread of the cloud on the scattering coefficient in comparison with the effect of thermal eddies at high particle concentrations.

The dynamics of the motion of the carrier medium are greatly altered when the particle concentration is high, since the precipitation and thermal convection processes are separated in time. The speed of fall of the cloud under the action of the external force is high owing to the great gasdynamic interaction between the particles and, hence, the particles are precipitated on the plane $y = 0$ before the onset of thermal convection. The two symmetric precipitation eddies formed when the cloud falls exist close to the precipitation surface for a long time after the precipitation of the particles. Since the particle concentration and, hence, the gas temperature are highest at the cores of the descending eddies, regions of gas with increased temperature are formed at sites of their interaction with the plane after precipitation of the particles. In these regions thermal convection occurs: The hot gas rises, causing the formation of symmetric thermal eddies on the right ($x > 0$) and left ($x < 0$) of the precipitation eddies. The ascending thermal eddies increase in size, their intensity increases, and the precipitation eddies gradually decay. As an example, we cite some results of calculation with the parameters of curve 4 and $\alpha_2^0 = 0.01$: The time for complete precipitation of the particles is 2.5, the time for onset of thermal convection is approximately four times greater, and the centers of the hot-gas regions after precipitation of the particles lie at $x \approx \pm 4.4$.

We have considered the situation where the intensity of the thermal eddies is not sufficient for the abstraction of a considerable number of particles. Calculations show that for fairly high α_2^0 and θ_p the vertical extension of the cloud terminates in its breakup (division) into two smaller clouds, one of which moves downward towards the precipitation plane, while the other is carried upwards by the flow of hot gas. We can expect that the behavior of the precipitation curves and the scattering coefficient in this case will differ significantly from the relations shown in Figs. 2-4. The precipitation of the particles will take place in two stages: first from the bottom cloud and then from the upper cloud (after it has cooled).

We note in conclusion that the features of precipitation of a cloud of hot particles examined in this paper are of interest from the viewpoint of the most diverse effects (aerosol clouds accompanying the eruption of volcanos, clouds of combustion products in fires, clouds of waste gases of industrial enterprises and in chemical reactors). The results obtained can be used to describe the aerodynamics of a cloud of slowly burning (combustion time less than the precipitation time) particles above a horizontal plane.

NOTATION

t, time; x, y, dimensionless coordinates with unit vectors i and j; ρ, U, θ, dimensionless density, velocity, and temperature; P, dimensionless gas pressure; f, force of interphase interaction; q, rate of interphase heat transfer; R, initial radius of cloud; H, dimensionless initial height of cloud; g, acceleration of external force; ρ_{10}, gas density at precipitation plane at initial time; T_0 and T_p, initial gas and particle temperatures; $\theta_p = (T_p - T_0)/T_0$, dimensionless initial temperature difference between phases; R_0, gas constant; n_0, maximum initial particle concentration; n, dimensionless particle concentration; n_s, particle concentration distribution on precipitation plane; N_s, particle concentration distribution on precipitation plane when $t \to \infty$; d_i/dt, substance derivatives; ρ_2^0, true particle density; α_2, volume fraction of particles; α_2^0, maximum volume fraction of particles at initial time; d, particle diameter; c_v and c_p, specific heats of gas at constant volume and pressure; η, dynamic viscosity; λ, thermal conductivity; c_2, specific heat of particles; c_d, resistance coefficient; M, Re, Pr, Nu – Mach, Reynolds, Prandtl, and Nusselt numbers; Re_p, variable Reynolds number of particle; Re_p^0, Reynolds number of particle based on characteristic convective velocity; ε, ratio of true densities of gas and particles; δ, ratio of particle diameter to cloud radius; m, scattering coefficient of cloud on precipitation surface; σ, relactive increase in radius of cloud due to its spread; $\langle \Theta \rangle$, dimensionless mean temperature after spread of cloud.

LITERATURE CITED

1. L. M. Moroz and Ya. I. Frenkel', "Hydrodynamics of drops of a suspension falling in a pure dispersion medium," Kolloidn. Zh., 11, No. 3, 178-183 (1949).
2. N. A. Fuks, Mechanics of Aerosols [in Russian], Izd. Akad. Nauk SSSR, Moscow (1955).
3. V. G. Khorguani, "Nature and speed of fall of a system of equal-sized particles," Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana, 11, No. 4, 394-401 (1966).
4. G. M. Makhviladze and O. I. Melikhov, "Numerical investigation of the fall of an assembly of monodisperse particles on a flat horizontal surface," Preprint No. 191, IPM Akad. Nauk SSSR, Moscow (1981).
5. G. M. Makhviladze and O. I. Melikhov, "Motion of an assembly of particles under the action of gravity and its precipitation on a flat horizontal surface," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 6, 64-73 (1982).
6. R. I. Nigmatulin, Fundamentals of Mechanics of Heterogeneous Media [in Russian], Nauka, Moscow (1978).
7. G. M. Makhviladze and S. B. Shcherbak, "A difference scheme for numerical investigation of unsteady two-dimensional motions of a compressible gas," Preprint No. 113, IPM Akad. Nauk SSSR, Moscow (1978).
8. G. M. Makhviladze and S. B. Shcherbak, "A numerical method of investigating unsteady three-dimensional motions of a compressible gas," Inzh.-Fiz. Zh., 38, No. 3, 528-535 (1980).
9. A. A. Samarskii, Theory of Difference Schemes [in Russian], Nauka, Moscow (1967).
10. R. S. Scorer, Environmental Aerodynamics, Horwood (1978).